

# Perfect fluid and $F(T)$ gravity descriptions of inflationary universe and comparison with observational data

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## Abstract

We describe in this paper the observables of inflationary models, in particular the spectrum index of torsion scalar perturbations, the tensor-to-scalar ratio, and the running of the spectral index, in the framework of perfect fluid models and  $F(T)$  gravity theories through the reconstruction methods. Then, our results on the perfect fluid and  $F(T)$  gravity theories of inflation are compared with recent cosmological observations such as the Planck satellite and BICEP2 experiment. Our studies prove that the perfect fluid and  $F(T)$  gravity models can reproduce the inflationary universe consistent above all with the Planck data. We have reconstructed several models and considered others which give the best fit values compatible with the spectral index of curvature perturbations, the tensor-to-scalar ratio, and the running of the spectral index within the allowed ranges suggested by the Planck and BICEP2 results.

Keywords: Scalar field, inflation, Slow-roll, e-folds.

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# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Scalar torsion and perfect fluid description of the Slow-roll parameters</b>	<b>5</b>
2.1	SLow-roll parameters . . . . .	5
2.2	Perfect fluid description of slow-roll parameters and observables . . . . .	7
2.3	Perfect fluid models reconstructions . . . . .	8
2.3.1	The linear form . . . . .	8
2.3.2	The exponential form . . . . .	9
2.3.3	Another form . . . . .	10
2.3.4	Comparison with the observations . . . . .	10
<b>3</b>	<b>Description of the SLOW-ROLL in <math>F(T)</math> gravity</b>	<b>12</b>
3.1	Observables of inflationary models in terms of the quantities in $F(T)$ gravity theories . . . . .	12
3.2	Reconstruction of $F(T)$ gravity models . . . . .	12
3.2.1	The linear form . . . . .	13
3.2.2	The exponential form . . . . .	14
3.3	Power-law model of $F(T)$ gravity . . . . .	15
<b>4</b>	<b>Conclusion</b>	<b>16</b>
	Bibliographie	17

## 1 Introduction

The recent data taken by the BICEP2 experiment ( Ade et al. 2014) on the tensor-to-scalar ratio of the primordial density perturbations, additionally to the observations by the satellites of the Wilkinson Microwave Anisotropy Probe (WMAP) ( Spergel et al. 2003; Hinshaw et al. 2013) and the Planck( Ade et al. 2014) have begotten many reflexion on inflation. The potential form of inflaton is related to the spectrum of the density perturbations generated during inflation ( Lidsey et al. 1997). Several models of inflation have recently been constructed such as quantum cosmological perturbations for predictions and observations ( Mukhanov 2013),

and others to account for the Planck and BICEP2 experiment ( Hazra et al. 2014). It exists some of them which are related to scalar field additionally to modified gravities models of inflation in comparison with the data analysis of the BICEP2 ( Joergensen et al. 2014; Gao and Gong 2014; Bamba et al. 2014).

Many studies with Various interesting results, have been done on the reconstruction of inflationary models in the framework of perfect fluid ,  $F(R)$  gravity and others modified gravity theories. Furthermore, the reconstruction of  $F(R)$  gravity models from observational data has been executed in ( Starobinsky 1980). It has been also done in supergravity ( Ferrara et al. 2014) and the related models ( Chakravarty and Mohanty 2015). All these realizations have been the attempts to make modified gravity models to explain the Planck and BICEP2 results ( Bamba et al. 2014; Pallis 2014 ). Moreover, Bamba and his collaborators have recently and explicitly performed the reconstruction of scalar field theories with inflation leading to the theoretical consequences compatible with the observational data obtained from the Planck and BICEP2 in terms of the spectral index of the curvature fluctuations, the tensor-to-scalar ratio, and the running of the spectral index in ( Bamba et al. 2014). They have also made the perfect fluid and  $F(R)$  gravity descriptions of inflation and its comparison with observational data in ( Bamba et al. 2014). In this last one , they have re-expressed the observables of inflationary models, i.e., the spectral index  $n_s$  of curvature perturbations, the tensor-to-scalar ratio  $r$ , and the running of the spectral index  $\alpha_s$  , in terms of the quantities in perfect fluid models and  $F(R)$  gravity theories. They have investigated several  $F(R)$  gravity models, especially, a power-law model which gives the best fit values compatible with the spectral index and tensor-to-scalar ratio within the allowed ranges suggested by the Planck and BICEP2 results. Moreover, the spectral index's features have been studied in induced gravity ( Kaiser 1994) and scalar-tensor theories ( Kaiser 1995). **An important work which explains a scenario of inflation for the first time in the framework of teleparallel gravity preciously in the modified version  $F(T)$  gravity is proposed in (Jamil et Al.). Their investigation gived a value of Spectral scalar index  $n_s$  which is compatible in a reasonable agreement with the WMAP7 and for pre Planck  $n_s < 0.961$ .**

In parallel way to these works with interesting results on the inflationary models in the framework of perfect fluid and modified gravities, we investigated in this paper, the descriptions of inflation through perfect fluid and  $F(T)$  gravity models. We reformulate the observables of

inflationary models in terms of the quantities of scalar torsion and in perfect fluid and  $F(T)$  gravity models and compare the theoretical representations with recent data from Planck et BICEP2. We here emphasize that the  $F(T)$  gravity coupling with scalar field theory have already been explored in several interesting cosmological works the density perturbation growth in teleparallel cosmology ( Geng and Wu 2013) and dynamical features of scalar-torsion theories ( Skugoreva et al. 20015). This present work wants to explain the importance of formulations of the observables for inflationary models in terms of the perfect fluid and  $F(T)$  gravity. It is physically motivated by the fact that in ordinary scalar field model of inflation, the spectral index, the tensor-to-scalar ratio, and the running of the spectral index are represented by using the potential  $V(\phi)$  of the scalar field. Consequently, scalar field models are consistent with the observations and by comparing the theoretical representations of theses observables with observations, we can get information on the properties of the perfect fluid and  $F(T)$  gravity models to account for the observations in the early universe. Our approach in terms of inflation can permit to find the conditions in which the perfect fluid and  $F(T)$  gravity models can be viable from the cosmological point of view. We normalize to unit the following constants  $k_B = c = \hbar = 1$  and express the gravitational constant  $8\pi G_N$  by  $\kappa^2 \equiv 8\pi/M_{Pl}^2$  with Planck mass of  $M_{Pl}^2 = G_N^{-1/2}$ .

The plan of the manuscript is outlined as follows: In Sec.2 we write the slow-roll parameters and express the observables for inflationary models in the first time as function of scalar torsion and in the second time in perfect fluid description. Then, in Sec.3 we re-express these parameters and observables in framework of  $F(T)$  gravity. All ours investigations have been ended by comparison with the Planck and BICEP2 data. Finally, the Sec.4 is devoted to the conclusions. We present the explicit expressions of the slow-roll parameters the observables for inflationary models as function of scalar torsion and in the formulation of perfect fluid in Appendixes *A* and *B*, respectively. The theoretical representations of observables for inflationary models in  $F(T)$  gravity are presented in Appendix *C*. Appendix *D* is devoted to the representations of the observables for inflationary models in the linear form of the scalar torsion and in the exponential form. We have ended this work on Appendix *E* where we have reminded the relation between the equation of state (*EoS*) parameter and the tensor-to-scalar ratio.

## 2 Scalar torsion and perfect fluid description of the Slow-roll parameters

As we have mentioned it in our introduction, the slow-roll parameters are related to inflaton namely the potential of scalar field. In this section, we express these parameters by the scalar torsion and in terms of perfect fluid.

### 2.1 SLow-roll parameters

The action of teleparallel gravity coupled with the model of scalar field  $\phi$  is given by ( Geng and Wu 2013 )

$$S = \int \left( \frac{T}{2\kappa^2} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) e d^4x, \quad (1)$$

where  $T$  is the scalar torsion and  $e$  the determinant of tetrad  $e^a{}_\mu$ . The slow-roll parameters,  $\varepsilon$ ,  $\eta$  et  $\xi$  are defined by (Bamba et al. 2014)

$$\varepsilon \equiv \frac{1}{2\kappa^2} \left( \frac{V'(\phi)}{V(\phi)} \right)^2, \quad \eta \equiv \frac{1}{\kappa^2} \frac{V''(\phi)}{V(\phi)}, \quad \xi^2 \equiv \frac{1}{\kappa^4} \frac{V'(\phi)V'''(\phi)}{(V(\phi))^2}. \quad (2)$$

Here and for the rest, the prime means the derivative with respect to argument such as  $V'(\phi) \equiv \partial V(\phi)/\partial \phi$ , and others. For the scalar field models, the spectral index  $n_s$ , of curvature perturbations, the tensor-to-scalar ratio  $r$  of the density perturbations and the running of spectral index  $\alpha_s$  are expressed as (Bamba et al. 2014)

$$n_s - 1 \sim -6\varepsilon + 2\eta, \quad r = 16\varepsilon, \quad \alpha_s \equiv \frac{dn_s}{d\ln\kappa} \sim 16\varepsilon\eta - 24\varepsilon^2 - 2\xi^2. \quad (3)$$

The variation of action (1) with respect to the tetrad  $e^a{}_\mu$  gives ( Geng and Wu 2013; Skugoreva et al. 2015)

$$\begin{aligned} & \frac{1}{\kappa^2} \left[ \frac{1}{4} \delta_\beta^\rho T + T^\sigma{}_{\nu\beta} S_\sigma{}^{\rho\nu} + e^{-1} e^a{}_\beta \partial_\alpha (e e_a{}^\sigma S_\sigma{}^{\rho\alpha}) \right] = \\ & \frac{1}{2} \left[ \delta_\beta^\rho \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right) - \frac{1}{2} \delta_\beta^\mu g^{\mu\rho} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \delta_\beta^\nu g^{\nu\rho} \partial_\mu \phi \partial_\nu \phi \right]. \end{aligned} \quad (4)$$

We consider an universe described by the following flat Friedmann-Lemaître-Roberson-Walker FLRW metric:

$$ds^2 = -dt^2 + a^2(t) \sum_{i=1,2,3} (dx^i)^2. \quad (5)$$

Here,  $a(t)$  is the scale factor and  $H \equiv \dot{a}/a$  is the Hubble parameter. From (5), one obtains the torsion scalar in function of  $H$  by  $T = -6H^2$ . The point  $(.)$  means here the derivative with respect to time as  $\partial/\partial t$ .

We can now find out the expressions of the slow-roll parameters as function of torsion scalar. Indeed, by using the relations (4) and (5), one gets the field equation given by:

$$\frac{T}{2\kappa^2} = -\frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (6)$$

$$\frac{\dot{T}}{12H\kappa^2} = \frac{1}{2}\dot{\phi}^2. \quad (7)$$

We define the scalar field  $\phi$  by a new scalar field  $\varphi$  such as  $\phi = \phi(\varphi)$  and we identify  $\varphi$  to e.folds number  $N$ . If one defines  $N$  by scale factor as  $N \equiv \ln(a/a_0)$ , we obtain  $\dot{N} = \dot{\varphi} \equiv H(N)$ . The equations (6) and (5) become

$$\frac{T(N)}{2\kappa^2} = \frac{1}{12}w(\varphi)T(N) + V(\phi(\varphi)), \quad (8)$$

$$\frac{T'(N)}{12\kappa^2} = -\frac{1}{12}w(\varphi)T(N), \quad (9)$$

with  $w(\varphi) \equiv (d\phi/d\varphi)^2$ . So, if we consider the torsion scalar as function of  $N$ , by combining (8) and (9), one expresses  $w(\varphi)$  and  $V(\phi) \equiv V(\phi(\varphi))$  by  $N$  as

$$w(\varphi) = \left[ \frac{-T'(N)}{\kappa^2 T(N)} \right]_{N=\varphi}, \quad (10)$$

$$V(\varphi) = -\frac{1}{12\kappa^2} [6T(N) + T'(N)]_{N=\varphi}. \quad (11)$$

We find  $T = T(N)$  and  $\varphi = N$  as solution for field equation  $\phi$  or  $\varphi$  and Einstein equations because of equivalence between Teleparallel and General Relativity. We also find that  $T'(N) > 0$  because  $w(\varphi) > 0$  and  $T(N) < 0$ . **Thus, we can now express in terms of  $T(N)$  all the slow-roll parameters and the observables by making using the relations (10) and (11) in their basic expressions defined in (2) and (3) respectively. One gets**

$$\epsilon = -\frac{T(N)}{2T'(N)} \left( \frac{6T'(N) + T''(N)}{6T(N) + T'(N)} \right)^2 \quad (12)$$

$$\eta = -[6(T'(N))^3 + T''(N)(T'(N))^2 + 6T''(N)T'(N)T(N) - (T''(N))^2T(N)] \quad (13)$$

$$\begin{aligned} \xi^2 = & \frac{T(N)}{2T'(N)} \frac{6T'(N) + T''(N)}{(6T(N) + T'(N))^2} [7T''(N) + 3T'''(N) + \frac{2T''''(N)T(N)}{T'(N)} + \frac{6T'''(N)T(N)}{T'(N)} \\ & - \frac{4T'''(N)T''(N)(N)}{(T'(N))^2} + \frac{2(T''(N))^3T(N)}{(T'(N))^3} - \frac{6(T''(N))^2T(N)}{(T'(N))^2} - \frac{(T''(N))^2}{T'(N)}] \end{aligned} \quad (14)$$

$$\begin{aligned} n_s = & 1 + \frac{3T(N)(6T'(N) + T''(N))^2}{T'(N)(6T(N) + T'(N))^2} \\ & + \frac{-6T'(N)^3 - T'(N)^2T''(N) + T(N)T''(N)^2 - 2T(N)T'(N)(3T''(N) + T^{(3)}(N))}{T'(N)^2(6T(N) + T'(N))} \end{aligned} \quad (15)$$

$$r = -\frac{8T(N)(6T'(N) + T''(N))^2}{T'(N)(6T(N) + T'(N))^2}, \quad (16)$$

$$\begin{aligned} \alpha_s = & [T(N)(6T'(N) + T''(N))(T'(N)^4(144T'(N)^2 + 5T''(N)^2 + T'(N)(41T''(N) - 3T^{(3)}(N))) \\ & + 12T(N)^2T'(N)(-4T''(N)^3 + T'(N)T''(N)(9T''(N) + 8T^{(3)}(N)) + T'(N)^2(51T''(N) + 9T^{(3)}(N) \\ & - 2T^{(4)}(N))) - 2T(N)T'(N)^2(216T'(N)^3 + 6T''(N)^3 + 3T'(N)T''(N)(11T''(N) - 2T^{(3)}(N)) \\ & + T'(N)^2(150T''(N) - 3T^{(3)}(N) + T^{(4)}(N))) - 72T(N)^3(T''(N)^3 - T'(N)T''(N)(3T''(N) \\ & + 2T^{(3)}(N)) + T'(N)^2(3T^{(3)}(N) + T^{(4)}(N))))]/(T'(N)^4(6T(N) + T'(N))^4). \end{aligned} \quad (17)$$

## 2.2 Perfect fluid description of slow-roll parameters and observables

We rewrite here, the slow-roll parameters in framework of perfect fluid. According to the FLRW metric (5), the gravitational field equations for a perfect fluid take the following forms (Geng and Wu 2013 )

$$\frac{T(N)}{2\kappa^2} = -\rho(N), \quad (18)$$

$$\frac{T'(N)}{6\kappa^2} = \rho(N) + P(N), \quad (19)$$

where  $\rho(N)$  and  $P(N)$  are energy density and pressure of perfect fluid respectively. Suppose the following general equation of state:

$$P(N) = -\rho(N) + f(\rho), \quad (20)$$

with  $f(\rho)$  a function of  $\rho$ . In this case, the equation (19) takes the form

$$\frac{T'(N)}{6\kappa^2} = f(\rho). \quad (21)$$

The conservation law  $\rho'(N) + 3[\rho(N) + P(N)] = 0$  also becomes

$$\rho'(N) + 3f(\rho) = 0. \quad (22)$$

By combining equations (21) and (22), we obtain

$$\frac{T''(N)}{6\kappa^2} = -3\tilde{f}(\rho)f(\rho). \quad (23)$$

We emphasize here that the tilde of  $\tilde{f}$  means its derivative with respect to  $\rho$  i.e  $\tilde{f}(\rho) \equiv df(\rho)/d\rho$ , whereas the prime means the derivative with respect to  $N$  i.e  $T'(N) \equiv dT(N)/dN$  and  $\rho'(N) \equiv d\rho(N)/dN$ . **By using the relations (10), (11), (18) and (21), we can re-write the slow-roll parameters and the observables as only function of  $\rho(N)$  and  $f(\rho(N))$  as it was done in (Bamba et Al. 2014).**

## 2.3 Perfect fluid models reconstructions

We study two forms of scalar torsion to reconstruct two models of perfect fluid: the linear form and the exponential form.

### 2.3.1 The linear form

We investigate here the linear form of  $T(N)$  given by:

$$T(N) = D_0N + D_1, \quad (24)$$

with  $D_0 > 0$  and  $D_1 < 0$  constants. The physical motivation of this choice is that, according to exponential inflation of slow-roll, the scale factor is given by  $a = \bar{a} \exp(H_{int}t)$  where  $\bar{a}$  is a constant (Bamba et al. 2014).  $H_{int}$  is the Hubble parameter at the inflationary stage and it is approximately constant i.e it weakly depends of time. To express this weak time dependence of  $H$  and consequently of scalar torsion  $T$  we use the form in (24) where the e-fold number plays the role of time. In this case, if  $D_1/D_0 \ll N$ , the time dependence of scalar torsion during inflation is negligible. By using the relations (18), (19), one obtains:

$$\rho(N) = -\frac{1}{2\kappa^2}(D_0N + D_1), \quad P(N) = \frac{1}{6\kappa^2}[(3N + 1)D_0 + 3D_1]. \quad (25)$$



By eliminating  $N$  between these last equations, we find the following relation

$$P(N) = \frac{D_0}{6\kappa^2} - \rho(N), \quad (26)$$

which, added to the general equation of state (20), gives

$$f(\rho) = \frac{D_0}{6\kappa^2}. \quad (27)$$

From equations in (25), we can deduce the state equation of perfect fluid according to the linear form of scalar torsion as

$$\omega(N) = \frac{P(N)}{\rho(N)} = -1 + \frac{f(\rho)}{\rho(N)} = \frac{(3N+1)D_0 + 3D_1}{3(D_0N + D_1)}. \quad (28)$$

### 2.3.2 The exponential form

The second example of perfect fluid model studied in this work is whose scalar torsion is given by the following exponential function of  $N$

$$T(N) = D_2 N e^{\beta N} + D_3, \quad (29)$$

with  $D_2 > 0$ ,  $D_3 < 0$  and  $\beta > 0$  constants. The physical reason of the choice of this form is the following. During power law inflation, the scale factor is given by  $a = \bar{a} t^{\hat{p}}$  with  $\hat{p}$  constant. Then, the scalar torsion during inflation is so  $T = -6(\hat{p}/t)^2$  and becomes  $T = -6\hat{p}^2 \exp(-2N/\hat{p})$ . This last form is equivalent to (29) if we make  $D_2 = -6\hat{p}^2$ ,  $\beta = -2/\hat{p}$  and  $D_3 = 0$ . Such an exponential form can reproduce the power-low inflation.

By introducing relation (29) in the gravitational equations (18) and (19), one gets

$$\rho(N) = -\frac{1}{2\kappa^2}(D_2 N e^{\beta N} + D_3), \quad P(N) = \frac{1}{6\kappa^2}[(3 + \beta)D_2 N e^{\beta N} + 3D_3]. \quad (30)$$

Elimination of  $N$  between these equations gives

$$P(N) = -(1 + \frac{\beta}{3})\rho(N) - \frac{\beta D_3}{6\kappa^2}. \quad (31)$$

From equation (20), one obtains:

$$f(\rho) = \frac{\beta \rho(N)}{3} - \frac{\beta D_3}{6\kappa^2}. \quad (32)$$

Then, we find the parameter of state of perfect fluid model corresponding to the exponential form

$$\omega(N) = \frac{(3 + \beta)D_2Ne^{\beta N} + 3D_3}{3(D_2Ne^{\beta N} + D_3)}. \quad (33)$$

### 2.3.3 Another form

We consider here a model studied in (Mukhanov 2013) whose state parameter is given by

$$\omega(N) = -1 + \frac{\bar{\beta}}{(1 + N)\bar{\gamma}}, \quad (34)$$

with  $\bar{\beta}$  and  $\bar{\gamma}$  free parameters. By solving the system of equations formed by (18), (19) et (34), we obtain

$$T = \bar{T} \exp \left[ \frac{-3\bar{\beta}(1 + N)^{1-\bar{\gamma}}}{1 - \bar{\gamma}} \right] \quad (35)$$

where  $\bar{T}$  is constant. We can now specify the expressions of some observables as example the spectral index and the tensor-to-scalar ratio by introducing (35) in (15) and (16)

$$n_s = \frac{1}{3(-2(1 + N)^\gamma + \beta)^2} (1 + N)^{-2-\gamma} [-9(1 + N)^2\beta^3 + 3\beta^2(1 + N)^{1+\gamma}(13 + 13N - \gamma) - 2(1 + N)^{2\gamma}\beta(24(1 + N)^2 - \gamma + \gamma^2) + 2(1 + N)^{3\gamma}(6 + 6N^2 - (-4 + \gamma)\gamma + 6N(2 + \gamma))], \quad (36)$$

$$r = \left( \frac{8(1 + N)^{-2-\gamma}\beta(3(1 + N)\beta + (1 + N)^\gamma(-6 - 6N + \gamma))^2}{3(-2(1 + N)^\gamma + \beta)^2} \right). \quad (37)$$

We deduce that for the appropriate value of parameters  $\{\bar{\beta}, \bar{\gamma}\}$ , we can get the values of observables  $n_s$  and  $r$ .

### 2.3.4 Comparison with the observations

Suppose that the time variation of  $f(\rho)$  et  $\rho$  during inflation is sufficiently small and that inflation is almost exponential as  $\omega(N) \equiv P(N)/\rho(N) = -1 + f(\rho)/\rho(N) \approx -1$ , i.e.,  $|f(\rho)/\rho(N)| \ll 1$ , one gets from expressions obtained in subsection 2.2 after writting the slow-roll parameters and the observables in term of perfect fluid, the following approximatif relations.

$$n_s \sim 1 - 6 \frac{f(\rho)}{\rho(N)}, \quad r \approx 24 \frac{f(\rho)}{\rho(N)}, \quad \alpha_s \approx -9 \left( \frac{f(\rho)}{\rho(N)} \right)^2. \quad (38)$$

We present here the recent observations on spectral index  $n_s$ , the tensor-to-scalar ratio  $r$  and the running of spectral index  $\alpha_s$ . The recent data of Planck satellite ( Ade et al 2014) suggested  $n_s = 0.9603 \pm 0.0073(68\%CL)$ ,  $r < 0.11(95\%CL)$ , and  $\alpha_s = -0.0134 \pm 0.0090(68\%CL)$  [Planck et WMAP ( Spergel et al. 2003; Hinshaw et al. 2013)], whose negative sign is at  $1.5\sigma$ . The BICEP2 experiment (Ade et al. 2014) implies  $r = 0.20^{+0.07}_{-0.05}(68\%CL)$ . It is mentioned that discussions exist on how to subtract the foreground, for example in ( Ade et al 2015; Kamionkowski and Kovetz 2014 ). Recently, progress appear also in ( Colley and Gott 2015) to ensure the BICEP2 declarations. It has been also remarked that the representation of  $\alpha_s$  is also given in ( Bassett et al. 2006).

From equation (38), we can see that when the condition  $f(\rho)/\rho(N) = 6.6510^{-3}$  is realized at the inflationary era, we find  $(n_s, r, \alpha_s) = (0.960, 0.160, -3.9810^{-4})$ . In the case of linear form of  $T$  from equation (24), if  $D_1/D_0 \gg N$  and  $-D_0/(3D_1) = 6.6510^{-3}$ , the change of value of  $\omega(N)$  is consider to be negligible, and the above condition can be met at the inflationary stage. In the second time, for the exponential form with the condition  $\beta = 2.010^{-4}$  and then  $\beta N \ll 1$ , and that  $-1/3\{[1 + D_3/(D_{2,\beta})]\} = 6.6510^{-3}$ ,  $\omega(N)$  can be seen as a constant and above condition can be satisfied during inflation. As consequence, the conclusion is that perfect fluid can lead to Planck results with  $r = \mathcal{O}(0.1)$ , which value is compatible with BICEP2 experiment.

Thus, we mention here some concrete perfect fluid models ( Astashenok et al. 2012) . For model of  $P(N) = -\rho(N) + f(\rho)$  with  $f(\rho) = \bar{f} \sin(\rho(N)/\bar{\rho})$  where  $\bar{f}$  is constant and  $\bar{\rho}$ , the fiducial value of  $\rho$  known to produce the scenario of Pseudo-Rip ( Frampon et al. 2011), if  $\rho(N)/\bar{\rho} \ll 1$  and  $f(\rho)/\rho(N) \approx \bar{f}/\bar{\rho} = 6.6510^{-3}$  behaves almost constant and the above condition can be satisfied. Then, We have examined another model of  $P(N) = -\rho(N) + f(\rho)$ , where  $f(\rho) = (\rho(n))^\tau$  with  $\tau(\neq 0)$  constant. By using equation (18), one gets  $f(\rho)/\rho(N) \approx (-T_{inf}/\kappa^2)^{\tau-1}$ , where the constant  $T_{inf}$  means the scalar torsion at the slow-roll inflation regime. So, similarly to the first example, when  $f(\rho)/\rho(N) \approx (-T_{inf}/\kappa^2)^{\tau-1} = 6.6510^{-3}$ ,  $f(\rho)/\rho(N)$  can be considered constant and the above condition can be met.

### 3 Description of the SLOW-ROLL in $F(T)$ gravity

We describe in this section, the slow-roll parameters in the framework of  $F(T)$  gravity.

#### 3.1 Observables of inflationary models in terms of the quantities in $F(T)$ gravity theories

The action in context of  $F(T)$  gravity theories is defined by

$$S_{F(T)} = \int e \left( \frac{F(T)}{2\kappa^2} + \mathcal{L}_{matter} \right) dx^4, \quad (39)$$

The equation of motion is determinated by (Salako et al 2013)

$$S_{\beta}{}^{\rho\alpha} \partial_{\alpha} T f_{TT} + [e^{-1} e^a{}_{\beta} \partial_{\alpha} (e e_a{}^{\sigma} S_{\sigma}{}^{\rho\alpha}) + T^{\sigma}{}_{\nu\beta} S_{\sigma}{}^{\rho\nu}] f_T + \frac{1}{4} \delta_{\beta}^{\rho} f = 4\pi \mathcal{T}_{\beta}^{\rho}, \quad (40)$$

where  $\mathcal{T}_{\beta}^{\rho}$  represents energy momentum tensor of matter. By using the metric  $FLRW$  in (5), the modified Friedmann equations take the forms

$$\frac{1}{2} F(T) - T F'(T) + \kappa^2 \rho_{matter} = 0, \quad (41)$$

$$\frac{1}{2} F(T) + (6H^2 + 2\dot{H}) F'(T) - 24H^2 \dot{H} F''(T) - \kappa^2 P_{matter} = 0. \quad (42)$$

$\rho_{matter}$  et  $P_{matter}$  are respectively energy density and the pressure of matter. The prime of  $F(T)$  means it derivative with respect to scalar torsion  $T$  as example  $F'(T) = \frac{dF(T)}{dT}$ ,  $F''(T) = \frac{d^2 F(T)}{dT^2}$ , etc. In vacuum, the equation (41) gives

$$T = \frac{F(T)}{2F'(T)}. \quad (43)$$

With the fact that the scalar tension  $T$  is function of  $N$ , by introducing the relation (43) in the equations (18) and (21), one gets  $\rho(N)$  and  $f(\rho)$  in the contest of  $F(T)$  gravity. All the slow-roll parameters and the observables can be expressed as function of  $F(T)$ .

#### 3.2 Reconstruction of $F(T)$ gravity models

We use the method of reconstruction as it is developed in (Nojiri and Odintsov 2006). We define the e-folds number as  $N \equiv -\ln(a/a_0)$  with  $a_0$  the scale factor at present time  $t_0$ , and the

redshift becomes  $z \equiv a_0/a - 1$ . Consequently, we have  $N = -\ln(1+z)$ . We write the scalar torsion in terms of  $N$  through the function  $D(N)$  as

$$T(N) = D(N) = D(-\ln(1+z)) \quad (44)$$

By using the relation (44) and the continuity equation  $\dot{\rho}_i + 3H(1+\omega_i)\rho_i = 0$ , the equation (42) can be rewritten as

$$\begin{aligned} 0 = & -3F(T) + [6D(N) + D'(N)] F'(T) + 2D(N)D'(N)F''(T) \\ & + 6 \sum_i \rho_{matter_{0i}} \omega_i a_0^{-3(1+\omega_i)} \exp[-3(1+\omega_i)N(T)], \end{aligned} \quad (45)$$

where the last term is equal to the total pressure of matter  $P_{matter}$ . Here, the matters are supposed to be fluid labeled by  $<i>$  with the same equation of state  $\omega_i \equiv P_{matter_i}/\rho_{matter_i}$  where  $\rho_{matter_i}$  and  $P_{matter_i}$  are energy density and the pressure of  $i^{th}$  fluid and  $\rho_{matter_{0i}}$  a constant. It follows that for any form of scalar tensor i.e the function  $D(N)$ , the differential equation (45) can be resolved and the corresponding gravitational action  $F(T)$  (the model) which reproduces the expansion history described by scalar torsion via the Hubble parameter  $H$ . Moreover, for particular  $F(T)$  model, the function  $D(N)$  can also be obtained. Then, with the expressions of observables of inflationary models described in subsection 2.1, we can get the corresponding predictions on the inflation for this particular model  $F(T)$ .

### 3.2.1 The linear form

We use the linear form of scalar tensor  $T$  of the relation (24). This prove that the e-folds number can also be expressed in terms of the scalar torsion as  $N(T) = (T - D_1)/D_0$ . **The Friedmann equation (45), in the vacuum, becomes second order differential equation in  $T$  labeled by**

$$-3F(T) + (6T + D_0)F'(T) + 2TD_0F''(T) = 0, \quad (46)$$

**whose resolution gives**

$$F(T) = C_1\sqrt{-T} + C_2\sqrt{-T} \left( -\frac{2e^{\frac{3T}{D_0}}}{\sqrt{-T}} - \frac{2\sqrt{3\pi}\text{Erf}\left[\frac{\sqrt{3}\sqrt{-T}}{\sqrt{D_0}}\right]}{\sqrt{D_0}} \right). \quad (47)$$

$C_1$  and  $C_2$  constants of integration whereas  $\text{Erf}[z]$  is the Gauss integral distribution given by  $\text{erf}[z] = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$ . **We have consequently reconstructed the gravitational Lagrangian**

density which can generate the required expansion for any given expansion history  $T$  or for any given e-folds number  $N$ . To putting out the observables of inflationary models corresponding to the expansion history form considered, one puts the relation (24) in those defined by (15), (16) and (17) and gets

$$n_s = 1 + \frac{6D_0[12D_1 + D_0(-1 + 12N)]}{(D_0 + 6D_1 + 6D_0N)^2}, \quad (48)$$

$$r = -\frac{288D_0(D_1 + D_0N)}{(D_0 + 6D_1 + 6D_0N)^2}, \quad (49)$$

$$\alpha_s = -\frac{864D_0^2(D_1 + D_0N)[3D_1 + D_0(-1 + 3N)]}{(D_0 + 6D_1 + 6D_0N)^4}. \quad (50)$$

The inflationary phase must last enough to account for initial conditions problems as example the so called problem of horizon and flatness. The value of e-folds number at the end of inflation must be  $N_e \gtrsim 50$ . **The slow-roll parameters should be smaller than unity during inflation; whereas they became larger than or equal to unity at the end of inflation,  $N = N_e$ . As examples, if  $(N, D_0, D_1) = (50.0, 0.850, -95.0)$  and  $(60.0, 0.950, -115)$ , one gets from the previous relations (48), (49), and (50) the following value of the observables  $(n_s, r, \alpha_s) = (0.967, 0.130, -5.32 \times 10^{-4})$  and  $(0.967, 0.131, -5.45 \times 10^{-4})$  respectively. Thus, the Planck results for  $n_s$  with  $r = \mathcal{O}(0.1)$  can be reached.**

### 3.2.2 The exponential form

Here, we use the exponential form of the scalar torsion in (29). In this case, the relation between  $N$  and the scalar torsion is expressed by  $D_2 e^{\beta N} = T - D_3$ . **Then, the second order differential equation obtained from the Friedmann equation (45) in the vacuum according to this form of scalar torsion is given by:**

$$-3F(T) + (6T + \beta T - \beta D_3)F'(T) + 2T(\beta T - \beta D_3)F''(T) = 0. \quad (51)$$

**Its resolution gives**

$$F(T) = C_1 \sqrt{-T} - 2C_2 (-D_3 - T)^{-3/\beta} \left(1 + \frac{T}{D_3}\right)^{3/\beta} {}_2F_1 \left[ -\frac{1}{2}, \frac{3}{\beta}, \frac{1}{2}, \frac{-T}{D_3} \right], \quad (52)$$

with  $C_1$  and  $C_2$  the constants of integration and  ${}_2F_1(x_1, x_2, x_3; y)$ , is the hypergeometric function where  $x_i (i = 1, \dots, 3)$  are constants and  $y$  the variable. The corresponding observables of inflationary models for the exponential expansion history are given by

$$n_s = 1 + \frac{\beta(6 + \beta)\{-6D_3^2 + e^{N\beta}D_2[2D_3\beta + e^{N\beta}D_2(6 + \beta)]\}}{[6D_3 + e^{N\beta}D_2(6 + \beta)]^2}, \quad (53)$$

$$r = -\frac{8e^{N\beta}D_2(e^{N\beta}D_2 + D_3)\beta(6 + \beta)^2}{(6D_3 + e^{N\beta}D_2(6 + \beta))^2}, \quad (54)$$

$$\alpha_s = \left\{ e^{N\beta}D_2(e^{N\beta}D_2 + D_3)\beta^2(6 + \beta) \left[ 5e^{2N\beta}D_2^2(6 + \beta)^2 - 2e^{N\beta}D_2D_3(6 + \beta) \right. \right. \\ \left. \left. \times (-66 + \beta^2) + 12D_3^2[51 + 2\beta(9 + \beta)] \right] \right\} / [6D_3 + e^{N\beta}D_2(6 + \beta)]^4. \quad (55)$$

We deduce from these that for  $(N, D_2, D_3) = (50.0, 1.10, -10)$  and  $(60.0, 1.20, -15.0)$ , one gets  $(n_s, r, \alpha_s) = (0.9627, 6.89 \times 10^{-2}, -6.4 \times 10^{-4})$  and  $(0.9652, 5.83 \times 10^{-2}, -5.23 \times 10^{-4})$  respectively.

We recall here that our investigation tries to explain the inflation scenario in the framework of  $F(T)$  gravity coupled with scalar field as it was already done in a lot of works and also through another gravity (Bamba et al. 2014 ; Jamil et al. 2015 ). The obtained models in (47) and (52) represent the geometrical part of the initial model described by the action in (39). The reconstruction approach followed to obtain these models which consists to neglect all matter fields, can also be justified by the fact that in the inflationary era the inflation is driven by inflaton field  $\phi$  (Jamil et al. 2015). Furthermore, for appropriate choice of the parameters of these models, they can lead to the very interesting models studied in (Jamil et al. 2015).

### 3.3 Power-law model of $F(T)$ gravity

In this part of section, we choose one concrete model to describe the observables of inflationary models. the concerning model is the power-law model of  $F(T)$  gravity given by  $F(T) = \mu(-T)^n$  where  $\mu$  and  $n$  are constants. By introducing this expression in the Friedmann equation (45) in the vacuum, we get a first order differential equation in  $D(N)$  which solution is:

$$D(N) = T(N) = D_4 \exp\left(-\frac{3N}{n}\right), \quad (56)$$

with  $D_4$  a negative constant. This reconstructed form is equivalent to exponential form in (29) with the following conditions  $D_2 = D_4$ ,  $D_3 = 0$  and  $\beta = -3/n$ . This solution is valid for  $n \neq 0$ . The slow-roll parameters with respect to (56) are written as  $\epsilon = 3/(2n)$ ,  $\eta = 3/n$ , and  $\xi^2 = 9/n^2$ . Since these parameters are constants, if  $n \gg 1$ , one gets  $\epsilon \ll 1$ ,  $\eta \ll 1$  and  $\xi^2 \ll 1$  during inflation. These results prove the conditions of slow-rolling are realized in  $F(T)$  gravity.

## 4 Conclusion

We have described in this paper the observables of inflationary models. These observables are related to the slow-roll parameters which explicitly depend from the potential of inflaton. This justifies the choice of the scalar field in the description of the observables. We have found out the expressions of these observables as function of scalar torsion. This description has been done in the framework of scalar field model coupled with the Teleparallel gravity. We have chased our investigations on the perfect fluid description of slow-roll parameters. After expressing the slow-roll parameters and the observables in terms of quantities of perfect fluid, we have reconstructed two perfect fluid models according to the linear and exponential forms of scalar tensor and gives additional model. We have compared the results with observation. We have found that perfect fluid can lead to the Planck results with  $r = \mathcal{O}(0.1)$ .

We have ended our study on the  $F(T)$  gravity description of the slow-roll parameters. We have also reconstructed two models as we have done in the case of perfect fluid. By choosing appropriate values for the free parameters of the models, one gets results on the observables especially  $n_s$  and  $r$  which are compatible with the Planck data on these observables. The last studied model in this work, the Power-law model of  $F(T)$  gravity, has permitted to confirm the realization of the conditions of slow-rolling in the framework of  $F(T)$  gravity.



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